

available at [www.sciencedirect.com](http://www.sciencedirect.com)journal homepage: [www.elsevier.com/jbmt](http://www.elsevier.com/jbmt)

FASCIA SCIENCE AND CLINICAL APPLICATIONS: MATHEMATICAL FASCIAL MODELLING

## Mathematical model of fiber orientation in anisotropic fascia layers at large displacements

Hans Chaudhry, Ph.D.<sup>a</sup>, Roman Max, Ph.D.<sup>a,\*</sup>, Stecco Antonio, M.D.<sup>b</sup>, Thomas Findley, M.D., Ph.D.<sup>c</sup>

<sup>a</sup> Department of Biomedical Engineering, New Jersey Institute of Technology, Newark, NJ 07102, USA

<sup>b</sup> Department of Physical Medicine and Rehabilitation, University of Padua, Padua 35120, Italy

<sup>c</sup> Health Care Management, War-Related Illness and Injury Center, VA Medical Center, East Orange, NJ 07108, USA

Received 22 February 2011; received in revised form 18 April 2011; accepted 26 April 2011

### KEYWORDS

Fascial;  
Reinforcement;  
Collagen fibers;  
Re-orientation;  
Tension

**Summary** A mathematical model is developed to determine the relationship between stretch and the orientation of fibers in the fascia. The transversely isotropic stress-strain relation for large displacements valid for the human fascia reinforced by the collagen fibers is employed. The relation between the orientation of fibers in the un-deformed and deformed state depending upon the stretch is plotted. It is observed that for greater fiber angle orientation, the fibers are more resistant to reorientation as the fascia is stretched longitudinally. It is also concluded that the reinforced fascia will always be in tension as the stretch is applied. However, we suggest future research to resolve the tension and compression issues in fascia. © 2011 Elsevier Ltd. All rights reserved.

### Introduction

Classically, the deep fascia is classified as irregular, dense connective tissue which envelops the muscles (Fawcett et al., 1994; Russ and Dehoff, 2000; Young, 2006). More recent studies (Benjamin, 2009; Langevin et al., 2009; Stecco et al., 2007, 2008) provide important evidence of a specific organization of the deep fasciae. In particular (Stecco et al., 2009, 2008), have demonstrated that deep fasciae of the limbs is characterized by layers which slide

relative to each other. In each layer, collagen fibres bundles have a parallel disposition along one direction, and the orientation changes between two layers. The angle formed by the orientations of the fibers relative to each other in man was found to be 78 degrees in the horizontal plane (Benetazzo et al., 2011). This is identical to the angle in the cow reported by Purslow (2010).

The matrix of the fascia which surrounds the collagen fibers is thought to consist of a proteoglycan gel (Purslow, 1989) with negligible modulus of elasticity in tension relative to the collagen fibers with modulus of elasticity as high as 1GPa (Fung, 1981). The angle of fiber orientation varies in endomysium (a type of fascia) from 2.5 degrees to 87.5 degrees. The endomysium is a layer of connective tissue that wraps muscle and is composed mostly of reticular

\* Corresponding author. Tel.: +1 973 596 5270; fax: +1 973 596 5222.

E-mail address: [max.roman@njit.edu](mailto:max.roman@njit.edu) (R. Max).

fibers. The weighted mean angle of orientation is found to be 59.1 degrees. Also, as the sarcomere length of the muscle increases, the mean fiber angle also increases (Purslow and Trotter, 1994).

A simple mechanical model of the perimysium (another type of fascia) was presented by Krenchel (1964) to find the mechanical properties of fiber networks, i.e. the fibers together with the sheet containing the fibers. The perimysium is a sheath of connective tissue that groups individual muscle fibers (anywhere between 10 and 100 or more) into bundles or fascicles. The elastic modulus of the network was calculated by a sum of the elastic modulus of the sheet plus that of the fibers and adjusted for angle orientation. Diamant et al. (1972) also applied finite beam-bending theory to describe the fiber deformation on extension. Purslow (1989) later made use of these models to predict the stress-strain relations at varying values of fiber orientation. From these results Purslow (1989) further found relationships between the relative stiffness of the perimysial network and stress versus the sarcomere lengths. Cox (1952) developed a mathematical model to consider the elasticity and strength of paper and other fibrous materials in the planar sheet for isotropic material. But, none of these models capture the non-linear (i.e. large deformation) behavior of the fibers during stretching of the fascia sheet. However, Adkins and Rivlin (1955) developed a mathematical model to describe the large elastic deformation of isotropic material reinforced by inextensible cords. The non-linear stress strain relation in the form of strain energy function was used in their analysis. Later, Aspden (1986) developed a mathematical model to study the relation between structure and mechanical behavior of fiber-reinforced composite isotropic materials at large strain keeping in mind the relevance to biological material. Their findings predict the reorientation of fibers with strain, in reasonable agreement with preliminary experimental results from spinal ligaments. No assumption was made about stress transfer to the fibers. Aspden confined his analysis to the fibers in tension, not in compression. This aspect of compression will be discussed in this paper.

Furthermore, in all these models the material is assumed to be isotropic. Since the spatial orientation of the collagen fibers differs from layer to layer, the fascia assumes anisotropic characteristics, i.e. the mechanical response of a single layer differs if the layer is loaded along the direction of the collagen fibres or along another direction. That means the fascia is anisotropic. We develop a mathematical model of the anisotropic fascia to further the understanding of the mechanical response during stretching in fascial manipulation treatment.

Therefore, a model that addresses the limitations of the above models is clearly needed. In view of this, the fascia is considered to be transversely isotropic (Chaudhry et al., 2007) and the analysis in this paper is based on the large deformation of extensible fibers using the strain energy function approach as employed by Adkins and Rivlin (1955) for isotropic material. The transverse isotropy means that the material has a plane in which the material properties are the same in different directions but in the direction perpendicular to this plane, the material properties are different.

## Methods

### Statement of the problem

We consider one type of fascia (endomysium) to be a thin sheet of highly elastic transversely isotropic incompressible material in the rectangular system  $(x_1, x_2, x_3)$ . Although, muscle lies under the fascia and exerts off-plane forces on the fascia, these forces are not considered in this model as the directions and magnitudes of these forces are not known. The incompressible material implies that the volume remains constant before and after deformation (Purslow, 1989). Further, the sheet is reinforced by two sets of fibers oriented at an angle  $\pm \alpha$  relative to the  $x_1$  axis, Figure 1a, based upon the information obtained from actual orientation of fibers in a layer of fascia, Figure 1b. These fibers are straight, and are located in the plane  $x_3 = 0$ . Each fiber of the first set are parallel to each other and spaced at a distance  $d_1$  apart. Similarly, the second set of fibers are parallel and spaced at a distance  $d_2$  apart. These fibers are assumed to be part of the fascia so that both deform together. The sheet is subjected to a homogeneous strain with principal extension ratios  $\lambda_1, \lambda_2, \lambda_3$  in the directions of  $x_1, x_2, x_3$  respectively. These extension ratios are defined as the deformed length after stretch divided by the original length before the stretch. The angles  $\pm \alpha$  become  $\pm \beta$  after deformation.

Thus if a line element  $dS$  in the un-deformed state becomes  $dS_1$  in the deformed state, then we can write (Adkins and Rivlin, 1955)

$$\left(\frac{dS_1}{dS}\right)^2 = \lambda_1^2 l^2 + \lambda_2^2 m^2 + \lambda_3^2 n^2 \quad (2.1)$$

As the fibers are in-extensible and lie in the plane  $x_3 = 0$  in the directions,  $(l_1, m_1, 0)$ ,  $(dS_1/dS) = 1$ , we get from (2.1),

$$\lambda_1^2 l^2 + \lambda_2^2 m^2 = 1 \quad (2.2)$$

Using  $l_1^2 + m_1^2 = 1$ , and  $l_1 = \cos \alpha, m_1 = \sin \alpha$ , we get from (2.2), the condition

$$\lambda_1^2 \cos^2 \alpha + \lambda_2^2 \sin^2 \alpha = 1 \quad (2.3)$$

It may be noted that the angle  $\beta$  after deformation is related to angle  $\alpha$  by

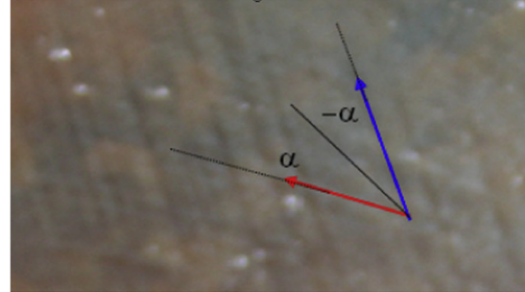
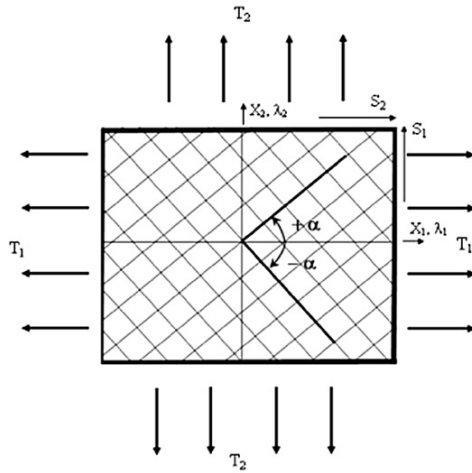
$$\cos \beta = \lambda_1 \cos \alpha, \quad \sin \beta = \lambda_2 \sin \alpha \quad (2.31)$$

We now suppose that the deformation is produced by a system of forces acting on the sheet uniformly around its edges, and parallel to the  $x_1, x_2$  axis so that stress resultants at any point of the sheet are given by:

1.  $(T_1, S_1)$  in the positive directions of  $x_1, x_2$ , respectively across a line parallel to the  $x_2$  axis.
2.  $(T_2, S_2)$  in the positive directions  $x_2, x_1$ , respectively across a line parallel to the  $x_1$  axis.

as shown in the figure above. It may be noted that since the force system is specified with respect to the principal directions of strain,  $S_1, S_2$  are the shear forces due to the tensions in the fibers only.

These forces are measured per unit length of the edges in the deformed state. Considering the elastic material and the system of fibers separately, we can write



**Figure 1** a) Rectangular element model of fascia subjected to forces along two perpendicular directions, i.e.  $X_1$  and  $X_2$  with two sets of parallel fibers orientated at  $\pm\alpha$ .  $\lambda_1, \lambda_2$  are principal extension ratios. b) Human fascial photograph from Stecco (2011).

$$T_1 = T'_1 + T''_1, \quad T_2 = T'_2 + T''_2 \quad (2.4)$$

where  $T'_1, T'_2$  are the forces required to deform the fascia without fibers, and  $T''_1, T''_2$  are the forces due to tension of the fibers in the fascia.

Now to determine  $T'_1, T'_2$ , we apply the stress–strain relation that is valid for the incompressible transversely isotropic material subject to large deformation, i.e. non-linear (Green and Adkins, 1970) since the fascia is assumed to be transversely isotropic (Chaudhry et al., 2007). The non-linear stress–strain relation expressed in the form of strain energy function  $W$  is given by Green and Adkins (1970):

$$\tau^{ij} = \phi g^{ij} + \psi B^{ij} + \rho G^{ij} + \theta M^{ij} + \lambda N^{ij} \quad (2.5)$$

where  $\tau^{ij} = (i, j = 1, 2, 3)$  are the stress components,

$$\begin{aligned} \phi &= 2 \frac{\partial W}{\partial I_1}, \quad \psi = 2 \frac{\partial W}{\partial I_2}, \quad \theta = \frac{\partial W}{\partial K_1}, \quad \lambda = \frac{\partial W}{\partial K_2}, \\ B^{ij} &= I_1 g^{ij} - g^{ir} g^{rs} G_{rs} \\ M^{ij} &= \frac{\partial \theta^i}{\partial x^3} \frac{\partial \theta^j}{\partial x^3} \end{aligned} \quad (2.6)$$

$$N^{ij} = \left( \frac{\partial \theta^i}{\partial x^3} \frac{\partial \theta^j}{\partial x^\alpha} + \frac{\partial \theta^i}{\partial x^\alpha} \frac{\partial \theta^j}{\partial x^3} \right) e_{\alpha 3}$$

$$e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_r}{\partial x_i} \frac{\partial u_r}{\partial x_j} \right)$$

$W$  is the strain energy function,  $W = W(I_1, I_2, I_3, K_1, K_2)$ , ( $I_3 = 1$ ) for incompressible material and

$$\begin{aligned} g_{ij} &= \frac{\partial x^r}{\partial \theta^i} \frac{\partial x^r}{\partial \theta^j}, \\ G_{ij} &= \frac{\partial y^r}{\partial \theta^i} \frac{\partial y^r}{\partial \theta^j}, \\ g^{ir} g_{rj} &= \delta^i_j, \quad G^{ir} G_{rj} = \delta^i_j \end{aligned} \quad (2.7)$$

$$I_1 = g^{rs} G_{rs}$$

$$I_2 = g_{rs} G^{rs} I_3$$

$$I_3 = G/g, \quad g = |g_{ij}|, \quad G = |G_{ij}|$$

$$K_1 = e_{33}, \quad K_2 = e_{3\alpha} e_{3\alpha}$$

where  $I_3 = 1$  for an incompressible material. It may be noted that a point  $P_0$  initially at  $x_i$  referred to a fixed rectangular cartesian coordinate system moves to  $y_i$  in the same coordinate system.  $\theta^i$  is defined as a general curvilinear coordinate system. It can be made to coincide with un-deformed or deformed state.  $e_{ij}$  are strain components.

### 3-D stresses due to the elastic matrix material without fibers

From the geometry of the deformation defined above, we write

$$y_1 = \lambda_1 x_1, \quad y_2 = \lambda_2 x_2, \quad y_3 = \lambda_3 x_3 \quad (3.1)$$

where any point  $(x_1, x_2, x_3)$  in the un-deformed state goes to  $(y_1, y_2, y_3)$  in the deformed state, and

$$\theta_1 = x_1 = x, \quad \theta_2 = x_2 = y, \quad \theta_3 = x_3 = z$$

Thus the displacements are given by:

$$\begin{aligned} u_1 &= (y_1 - x_1) = (\lambda_1 - 1)x_1 \\ u_2 &= (y_2 - x_2) = (\lambda_2 - 1)x_2 \\ u_3 &= (y_3 - x_3) = (\lambda_3 - 1)x_3 \end{aligned} \quad (3.2)$$

Using (3.2) into  $e_{ij}$  given in (2.6) we have

$$\begin{aligned} e_{11} &= (\lambda_1 - 1) + \frac{1}{2}(\lambda_1 - 1)^2 \\ e_{22} &= (\lambda_2 - 1) + \frac{1}{2}(\lambda_2 - 1)^2 \\ e_{33} &= (\lambda_3 - 1) + \frac{1}{2}(\lambda_3 - 1)^2 \end{aligned} \quad (3.3)$$

$$e_{12} = e_{23} = e_{31} = 0$$

Using (2.6) and (2.7) we find

$$g_{ij} = g^{ij} = \delta_{ik} \quad \text{and} \quad g = 1$$

$$G_{ij} = \begin{bmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{bmatrix} \quad (3.4)$$

$$G^{ik} = \begin{bmatrix} \lambda_1^{-2} & 0 & 0 \\ 0 & \lambda_2^{-2} & 0 \\ 0 & 0 & \lambda_3^{-2} \end{bmatrix} \quad (3.5)$$

$$B^{ik} = \begin{bmatrix} \lambda_2^2 + \lambda_3^2 & 0 & 0 \\ 0 & \lambda_3^2 + \lambda_1^2 & 0 \\ 0 & 0 & \lambda_1^2 + \lambda_2^2 \end{bmatrix}$$

$M^{ij} = 0$  for all  $i, j$  except for  $i = j = 3$ , i.e.  $M^{33} = 1$ . Similarly,  $N^{ij} = 0$  for all  $i, j$  except for  $i = j = 3$ , i.e.  $N^{33} = 2e_{33} = 2[(\lambda_3 - 1) + (1/2)(\lambda_3 - 1)^2]$

Using (3.4) in (2.7) gives the strain invariants

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \quad I_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2, \quad I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2 \quad (3.6)$$

To calculate the physical components of stress  $t_{ij}$  from the tensor components  $\tau^{ij}$  we make use of the following formula (Green and Zerna, 1968) given by

$$t_{ij} = \sqrt{\frac{G_{jj}}{G_{ii}}} \tau^{ij} \quad (3.7)$$

Thus, the physical components of stresses are given by

$$\begin{aligned} t_{11} &= \lambda_1^2 \tau^{11} = \lambda_1^2 \Phi + \lambda_1^2 (\lambda_2^2 + \lambda_3^2) \Psi + p \\ t_{22} &= \lambda_2^2 \tau^{22} = \lambda_2^2 \Phi + \lambda_2^2 (\lambda_3^2 + \lambda_1^2) \Psi + p \\ t_{33} &= \lambda_3^2 \tau^{33} = \lambda_3^2 \Phi + \lambda_3^2 (\lambda_1^2 + \lambda_2^2) \Psi + p \\ &\quad + \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A \end{aligned} \quad (3.8)$$

Using  $I_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2$  from (3.6) into (3.8) we can write

$$\begin{aligned} t_{11} &= \lambda_1^2 \Phi - (\lambda_2^2 \lambda_3^2) \Psi + p' \\ t_{22} &= \lambda_2^2 \Phi - (\lambda_3^2 \lambda_1^2) \Psi + p' \\ t_{33} &= \lambda_3^2 \Phi - (\lambda_1^2 \lambda_2^2) \Psi + p' + \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A \end{aligned} \quad (3.9)$$

where  $p' = p + \Psi I_2$ .

As the sheet is incompressible we can use  $I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2 = 1$ . Then (3.8) becomes

$$\begin{aligned} t_{11} &= T'_1 = \lambda_1^2 \Phi - \frac{1}{\lambda_1^2} \Psi + p' \\ t_{22} &= T'_2 = \lambda_2^2 \Phi - \frac{1}{\lambda_2^2} \Psi + p' \\ t_{33} &= T'_{33} = \lambda_3^2 \Phi - \frac{1}{\lambda_3^2} \Psi + p' + \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A \end{aligned} \quad (3.10)$$

As the  $x_1, x_2$  plane is free from normal traction we have  $T'_{33} = 0$ . Using this in (3.10) we have

$$p' = -\lambda_3^2 \Phi + \frac{1}{\lambda_3^2} \Psi - \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A$$

Then, we have

$$\begin{aligned} T'_1 &= \lambda_1^2 \Phi - \frac{1}{\lambda_1^2} \Psi - \lambda_3^2 \Phi + \frac{1}{\lambda_3^2} \Psi - \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A \\ T'_2 &= \lambda_2^2 \Phi - \frac{1}{\lambda_2^2} \Psi - \lambda_3^2 \Phi + \frac{1}{\lambda_3^2} \Psi - \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A \end{aligned} \quad (3.11)$$

The above set of equations (3.11) represent the forces in the sheet for transversely isotropic sheet without fibers subject to large deformation.

## 2-D stresses with fibers under tension

If the set of fibers carrying a tension  $t_1$  each after deformation make an angle  $\beta$  with the  $x_1$  axis and are spaced a distance  $d_1$  apart before deformation, the number of fibers required to cut a unit length in  $x_1$  and  $x_2$  directions are  $\sin\alpha/d_1$ , and  $\cos\alpha/d_1$ . These quantities change to  $\sin\alpha/\lambda_1 d_1$ , and  $\cos\alpha/\lambda_2 d_1$ . The tensions and spacing in the other set of fibers making an angle  $-\alpha$  with the  $x_1$  axis are denoted by  $t_2$  and  $d_2$ , respectively. Applying similar consideration to the second set of fibers and using the relation for the angle  $\beta$  after deformation and using (2.3)<sub>1</sub> we have as in Adkins and Rivlin (1955)

$$\begin{aligned} T'_1 &= \frac{\lambda_1}{\lambda_2} (\sigma_1 + \sigma_2) \cos^2 \alpha \\ T'_2 &= \frac{\lambda_2}{\lambda_1} (\sigma_1 + \sigma_2) \sin^2 \alpha \\ S_1 &= S_2 = \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\alpha \end{aligned} \quad (4.1)$$

where  $\sigma_1, \sigma_2 = ((t_1/d_1), (t_2/d_2))$ . We now assume that forces due to the elastic material without fibers (see section 3) can be added to the forces due to fibers tension (see section 4) linearly to find the resulting forces on the composite fascia of the sheet plus the collagen fibers.

Thus, using (3.11) and (4.1) in (2.4) the resultant forces on the reinforced fascia become

$$\begin{aligned} T_1 &= \lambda_1^2 \Phi - \frac{1}{\lambda_1^2} \Psi - \lambda_3^2 \Phi + \frac{1}{\lambda_3^2} \Psi - \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A \\ &\quad + \frac{\lambda_1}{\lambda_2} (\sigma_1 + \sigma_2) \cos^2 \alpha \\ T_2 &= \lambda_2^2 \Phi - \frac{1}{\lambda_2^2} \Psi - \lambda_3^2 \Phi + \frac{1}{\lambda_3^2} \Psi - \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A \\ &\quad + \frac{\lambda_2}{\lambda_1} (\sigma_1 + \sigma_2) \sin^2 \alpha \\ S &= \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\alpha \end{aligned} \quad (4.2)$$

## One dimensional extension stress combining fiber and elastic matrix

We assume that deformation is produced only by the tensile force  $T_1$ , then  $T_2 = S = 0$ , which means that  $\sigma_1 = \sigma_2$  and

$$\begin{aligned} \left\{ 2\lambda_3^2 (\lambda_3 - 1) + \lambda_3^2 (\lambda_3 - 1)^2 \right\} A &= \lambda_2^2 \Phi - \frac{1}{\lambda_2^2} \Psi - \lambda_3^2 \Phi + \frac{1}{\lambda_3^2} \Psi \\ &\quad + 2 \frac{\lambda_2}{\lambda_1} \sigma_1 \sin^2 \alpha \end{aligned} \quad (5.1)$$

Substituting (5.1) into  $T_1$  from (4.2) we have

$$T_1 = \left[ \Phi (\lambda_1^2 - \lambda_2^2) + \Psi \left( \frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2} \right) \right] + 2\sigma_1 \left( \frac{\lambda_1}{\lambda_2} \cos^2 \alpha - \frac{\lambda_2}{\lambda_1} \sin^2 \alpha \right) \quad (5.2)$$

**Results**

Equation (5.2) can be rewritten using (2.3) and (2.3)<sub>1</sub> with  $\lambda_2 \sim 1$  (due to extension only in the  $x_1$  direction) as

$$T_1 = \left[ \Phi(\lambda_1^2 - \lambda_2^2) + \Psi \left( \frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2} \right) \right] + 2\sigma_1 \frac{(2\lambda_1^2 \cos^2 \alpha - 1)}{\lambda_1} \quad (5.3)$$

The first term in (5.3) is positive since  $\lambda_1 > \lambda_2$ . For tensile force  $T_1$  in (5.3) to be positive, the second term will be positive if  $2\lambda_1^2 \cos^2 \alpha - 1 > 0$  or

$$\cos \alpha > \frac{1}{\lambda_1 \sqrt{2}} \quad (5.4)$$

Also, from (2.3)

$$\lambda_1 \cos \alpha < 1 \text{ or } \cos \alpha < \frac{1}{\lambda_1} \quad (5.5)$$

From (5.4) and (5.5), we find the condition for fascia to be in tension given by

$$\cos \alpha < \frac{1}{\lambda_1} \text{ and } \cos \alpha > \frac{1}{\lambda_1 \sqrt{2}} \quad (5.6)$$

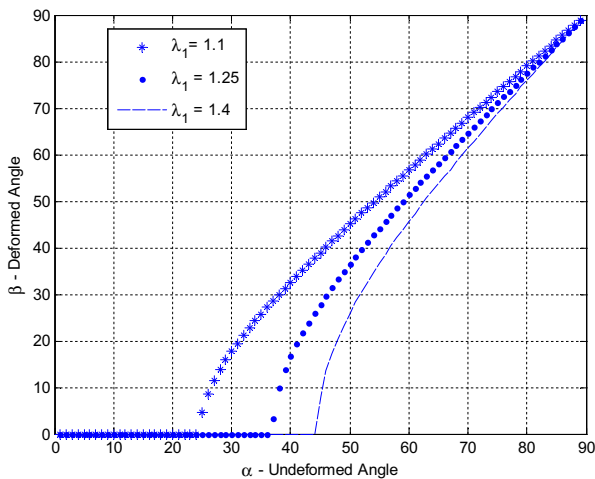
This can also be written in the form:

$$\cos^{-1} \frac{1}{\lambda_1} < \alpha < \cos^{-1} \frac{1}{\lambda_1 \sqrt{2}} \quad (5.7)$$

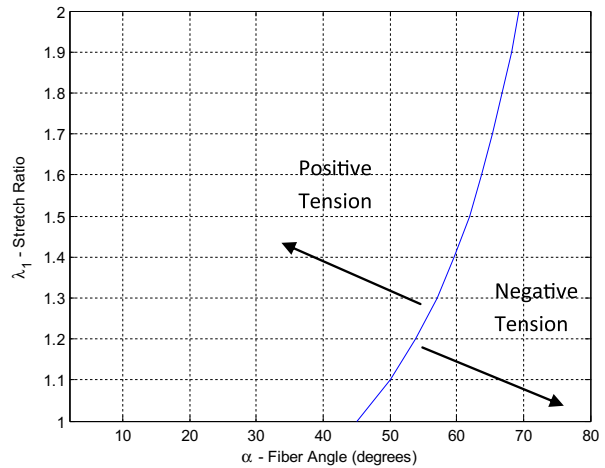
Using the condition (2.31) i.e.  $\cos \beta = \lambda_1 \cos \alpha$ , the plot between undeformed angle  $\alpha$  and deformed angle  $\beta$ , is plotted in Figure 2.

Again using the condition (5.7), the region of positive and negative tension in fibers is also plotted in the physiological range ( $1 \leq \lambda_1 \leq 2$ ) of stretch ratio (see Figure 3). It is noted from equation (5.3) that the total value of the positive and negative tension is always positive tension.

Figure 4 illustrates how the stretch along horizontal direction of the fascia for a given area results in a new shape of the same area making a smaller stretch in the



**Figure 2** Relation between undeformed and deformed angle after stretch for varying stretch ratios (i.e. deformed length over original length).



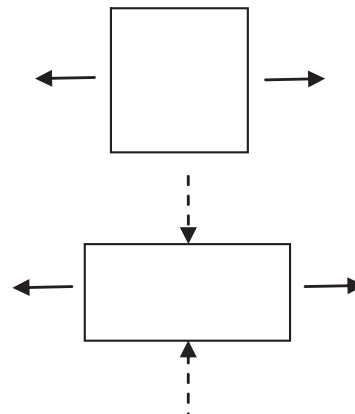
**Figure 3** Regions of positive and negative tensions in the fibers versus stretch ratio.

transverse direction. The significances of these plots (Figures 2–4) are discussed below in the conclusion section.

**Discussion**

To verify the predicted angles of orientation with our model compared to other models, we first find the predicted mean angle of fibril orientation of endomysium from constant shape model of Purslow and Trotter (1994) after a stretch of muscle sacromere length from its resting length of  $2 \mu\text{m}$  by 30%. The constant shape model is based on the assumption that as the muscle changes length at constant volume, the original cross-sectional shape of the fascicle remains constant, the cross section changes only in area.

It is found from (Purslow and Trotter, 1994) that the mean angle changes from 58 degrees to 43 degrees by the isoreal model, and from 58 to 48 degrees by the constant shape model. The isoreal is based on the inference that the



**Figure 4** Representative shape of fascia for a given area before and after stretch resulting in smaller stretch in the transverse direction compared to larger stretch in the longitudinal direction.

surface area as well as the volume of each muscle fiber remains constant as the muscle changes length. For our model, we employ the first equation of (2.31) to predict the change in angle as a result of 30% stretch. We find that the angle changes from 58 degrees to 47 degrees. These slight differences with other models can be due to manually analyzed orientation distribution in isoreal and constant shape models. However, the result of our constant volume model is very close to that of constant shape model (Purslow and Trotter, 1994).

Fascia may also have a key role in proprioception due to the presence of mechanoreceptors in the fascia which work primarily under the influence of tension in the fascia. The transmission of tensional forces to receptors is another fruitful area of investigation.

Fascia is generally considered as a structure which bears tensile and not compressive force. However, Ingber's group has elegantly demonstrated that intracellular structures such as DNA (Liedl et al., 2010) and microtubules (Ingber, 2010) can bear compressive forces. Also, fascia has a compartmentalized structure which if sealed may also support compression in a fashion similar to the air filled plastic bags commonly used as packaging materials.

## Limitations

In Figure 3, we find negative tension in a certain region. This may be due to our assumption by taking an element of fascia as isolated, when in fact it is not. Muscles are connected to the fascia beneath. The anatomical changes in the muscles beneath the fascia can come into play and may counteract the negative tension when the stretch reaches a certain threshold value. Further refinements of the model incorporation muscular connections of fascia is another topic for future research.

## Conclusion

A mathematical model has been successfully developed for highly elastic, transversely isotropic, incompressible material such as endomysium (a type of fascia) to establish the relationship between the orientation of the fibers and the stretch in the fascia. We also find a close prediction of the fiber orientation as a result of 30% stretch of endomysiums with our model compared to that of constant shape model of Purslow and Trotter (1994).

It is known that the connection between the fascia layers (Guimberteau) change the fiber orientation depending upon the applied forces or stretch. Therefore a relation between the fiber orientation before and after deformation which depends on the stretch is plotted (see Figure 2). The negative tension and positive tension threshold in the fibers produced as a result of stretch is shown in Figure 3. From equation (5.2), the first term of the total force  $T_1$  is always positive (in the absence of fibers) since the stretch in the longitudinal direction is always much greater than the resulting negative stretch (i.e. compression) in the transverse direction as demonstrated in Figure 4. Therefore, the total force in composite fascia (including fibers) must also be always positive after counteracting the negative tension in the fibers.

The above conclusion for fascia to be always in tension supports the key role of fascia for proprioception due to the presence of mechanoreceptors (in the fascia) which work only under the influence of tension in the fascia.

From Figure 2, we observe some interesting results. First, as the fibril angle increases, the fibers become more resistant to change after stretch of the fascia. Second, the deformed fibril angle becomes zero for fibers below a certain un-deformed fibril angle depending on the stretch. For example, for a stretch of 1.10, 1.25, 1.40, the threshold for un-deformed angles are 24, 36, and 44 degrees respectively, for which the deformed angles are zero. That means, below these fibril angles, the fibers get aligned with the horizontal direction. The clinical applications of these findings are as yet unexplored. This model may both suggest testable cellular mechanisms involved in fascial therapies, as well as modeling of specific therapies for testing improvements in clinical practice.

## Clinical relevance

Skin tension in the living being is not uniform in all directions, and these differences in direction (termed "Langers Lines") have long been used by surgeons to plan surgical incisions (Chaudhry et al., 1998). We are recently acquiring information on fascial fiber direction in individual fascial layers, as distinct from muscle fiber direction. In some manual therapies, the treatment forces are deliberately directed across the direction of the muscle fiber, as in "cross fiber" therapy. However, usually the practitioner determines force direction only by clinical judgement. This paper provides a means to model manual therapy interventions in longitudinal and transverse directions. This may allow more precise specification of manual therapy techniques.

## References

- Adkins, J.E., Rivlin, R.S., 1955. Large elastic deformations of isotropic materials, reinforcement by inextensible cords. *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences* 248 (944), 201–223.
- Aspden, R.M., 1986. Relation between structure and mechanical behavior of fiber-reinforced composite materials at large strains. *Proceedings of Royal Society of London A* 406, 287–298.
- Benetazzo, L., et al., 2011 January. 3D reconstruction of the crural and thoracolumbar fasciae. *Journal of Surgical Radiological Anatomy*. doi:10.1007/s00276-010-0757-7 (published on line).
- Benjamin, M., 2009. The fascia of the limbs and back- a review. *Journal of Anatomy* 214, 1–18.
- Chaudhry, H., et al., 2007. Mathematical analysis of applied loads on skeletal muscles during manual therapy. *Journal of American Osteopathic Association* 108 (12), 680–688.
- Chaudhry, H., et al., 1998. Optimal patterns for suturing wounds. *Journal of Biomechanics* 31 (7), 653–662.
- Cox, H.L., 1952. The elasticity and strength of paper and other fibrous materials. *British Journal of Applied Physics* 3, 72–79.
- Diamant, J., et al., 1972. Ultrastructure and its relation to mechanical properties as a function of aging. *Proceedings of Royal Society of London B* 18, 293–315.

- Fawcett, D.W., et al., 1994. *A Textbook of Histology*, 12th ed. Chapman & Hall, London.
- Fung, Y.C., 1981. *Biomechanics*. Springer, New York.
- Green, A.E., Adkins, J.E., 1970. *Large Elastic Deformations*, 2nd ed. Clarendon Press, Oxford.
- Green, A.E., Zerna, W., 1968. *Theoretical Elasticity*, 2nd ed. Clarendon Press, Oxford.
- Ingber, D.E., Mar 2010. From cellular mechanotransduction to biologically inspired engineering. *Annals of Biomedical Engineering* 38 (3), 1148–1161.
- Krenchel, H., 1964. *Fiber Reinforcement*. Akademisk Forlag, Copenhagen.
- Langevin, H.M., et al., 2009. Ultrasound evidence of altered lumbar connective tissue structure in human subjects with chronic low back pain. *BMC Musculoskeletal Disorder* 10, 151–160.
- Liedl, T., et al., 2010 Jul. Self-assembly of three-dimensional prestressed tensegrity structures from DNA. *Nature Nanotechnology* 5 (7), 520–524.
- Purslow, P.P., 2010 Oct. Muscle fascia and force transmission. *Journal of Bodywork & Movement Therapies* 14 (4), 411–417.
- Purslow, P., 1989. Strain-induced reorientation of an intramuscular connective tissue network: implications for passive muscle elasticity. *Journal of Biomechanics* 22 (1), 21–31.
- Purslow, P., Trotter, J.A., 1994. The morphology and mechanical properties of endomysium in series-fibred muscles: variations with muscle length. *Journal of Muscle Research and Cell Motility* 15, 299–308.
- Russ, J.C., Dehoff, R.T., 2000. *Practical Stereology*, second ed. Plenum Press, New York.
- Stecco, A., 2011. Personal Communication.
- Stecco, C., et al., 2007. Anatomy of the deep fascia of the upper limb. Second Part: study of innervation. *Morphologie* 91, 28–34.
- Stecco, C., et al., Aug 2009. Mechanics of crural fascia: from anatomy to constitutive modelling. *Surgical Radiology Anatomy* 31 (7), 523–529.
- Stecco, C., et al., 2008. Histological study of the deep fasciae of the limbs. *Journal of Bodywork and Movement Therapies* July 12 (3), 225–230.
- Young, B., 2006. *Wheater's Functional Histology: A Text and Colour Atlas*, fifth ed. Churchill Livingstone/Elsevier, Philadelphia.