

# Biological electric fields and rate equations for biophotons

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**Abstract** Biophoton intensities depend upon the squared modulus of the electric field. Hence, we first make some general estimates about the inherent electric fields within various biosystems. Generally, these intensities do not follow a simple exponential decay law. After a brief discussion on the inapplicability of a linear rate equation that leads to strict exponential decay, we study other, nonlinear rate equations that have been successfully used for biosystems along with their physical origins when available.

**Keywords** Biophotons · Electric fields · Nonlinear rate equations

## Introduction

Emission of biophotons from living cells and tissues of plant and animal origin is by now very well established (VanWijk 2001; Popp 1988; Popp et al. 1988; Rastogi and Pospisil 2010). Emission of biophotons can be technically considered as a type of chemiluminescence, although the emission of biophotons observed from biological tissues is much weaker than that observed in normal bioluminescence and also different from the thermal radiation emitted by tissues at their respective normal temperatures. It is widely accepted that reactive oxygen species initiate chain reactions leading to formation of excited species and

consequently biophoton emission (Cifra and Pospisil 2014; Pospisil et al. 2014).

In “[Electric fields and frequencies in biosystems](#),” we discuss the biological cell and the nucleus within as cavities for resonating electromagnetic fields with biophotons considered to reside initially in these cavities. Also, the frequencies and magnitude of the mean electric fields are estimated. In “[Discussion on nonexponential decay laws](#),” we discuss theoretical reasons for deviations from a linear rate equation and the resulting exponential behavior for the mean number of photons. It will be seen that, in almost all realistic situations, the mean number of photons does not obey an exponential decay law. In this regard, we mention in passing that detailed studies of the associated (Poisson and sub-Poisson) statistics that arise from nonlinear rate equations have also been made (Popp et al. 2002; Popp and Yan 2002; Chang 2008).

In “[Dynamics behind some simple nonlinear rate equations](#),” the nonlinear rate equations for various physical quantities such as the mean number of photons, size and mass of cells, etc. are motivated and the so-called logistic equations are discussed along with the dynamics generating them. In “[Growth of biological mass](#),” the growth of a biological mass is analyzed, and subsequently in “[Photons and cancer cell growth](#),” the biophoton intensity and its correlation with cancer cell growth is presented via some experimental results. In “[Luminescent decay](#),” hyperbolic decays are considered. We close the paper with some concluding remarks in “[Conclusions](#).”

## Electric fields and frequencies in biosystems

The original measurements of biophotons were reported first by Gurwitsch (1911, 1922, 1923) about a century ago.

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It was thought then that biophotons were emitted in the far-infrared and optical frequency ranges.

Let  $L$  denote the length scale of a cavity with a fundamental frequency  $\omega$  and a dielectric constant  $\varepsilon$ . These are related by

$$\omega = \left( \frac{\pi c}{L\sqrt{\varepsilon}} \right). \quad (1)$$

For a typical biological cell and nucleus we have, respectively, that

$$L_{\text{cell}} \sim 10^{-3} \text{ cm} \quad \text{and} \quad L_{\text{nucleus}} \sim 10^{-4} \text{ cm}. \quad (2)$$

It is therefore expected that

$$\omega_{\text{cell}} \sim \text{infrared} \quad \text{and} \quad \omega_{\text{nucleus}} \sim \text{optical}. \quad (3)$$

It is interesting to note here the size of the electric fields associated with one photon,

$$\left( \frac{\varepsilon E^2 L^3}{4\pi} \right) = \hbar\omega = \left( \frac{\hbar\pi c}{L\sqrt{\varepsilon}} \right), \quad (4)$$

$$E = \frac{2\pi}{L^2} \sqrt{\frac{\hbar c}{\varepsilon^{3/2}}}.$$

A cavity photon located in the cell and nucleus, respectively, has an associated electric field of

$$1 \text{ Gauss} \equiv 299.792458 \left( \frac{\text{V}}{\text{cm}} \right), \quad (5)$$

$$E_{\text{cell}}^{1 \text{ photon}} \sim 3.5 \times 10^{-2} \text{ Gauss} \sim 1 \left( \frac{\text{kV}}{\text{m}} \right), \quad (6)$$

$$E_{\text{nucleus}}^{1 \text{ photon}} \sim 350 \text{ Gauss} \sim 0.1 \left( \frac{\text{MV}}{\text{m}} \right).$$

While the photon frequencies of biological cavity modes in the cell and nucleus are in agreement with experiments, the estimates of the electric fields here presented are lower than previously reported (Popp 1988; Popp et al. 2000; Rattemeyer et al. 1981).

The relevance of our estimates for the average electric fields in a cell and the nucleus—considered as a cavity—for biology, resides in the fact that these fields are quite large, see “[Electric fields and frequencies in biosystems](#).” For example, the field due to one such biophoton in a nucleus is only slightly smaller than the breakdown field in humid conditions.

On the other hand, the magnitude of the electric field estimated above for the vacuum can be significantly modulated by variations in the effective local dielectric constant due to water, solvents, and neighboring material in general. Although such variations may have wide implications for biology, we shall not pursue this interesting avenue in the present paper.

## Discussion on nonexponential decay laws

It is often thought that the luminescence following an initial excitation should be exponential, but this invariably comes from some circular reasoning: either “this is well known to be the case” or “this is what one would expect from  $dN/dt = -\lambda N$ , where  $N$  is the excited population. Deviations from exponential decay are actually quite well known (Fonda et al. 1978), though relatively absent from most textbooks. Notable exceptions are the books by Ballantine (1990) and Merzbacher (1961). To quote from Merzbacher (op. cit. p. 513), “...the fact remains that the exponential decay law, for which we have so much empirical support in radioactive decay processes, is not a rigorous consequence of quantum mechanics but the result of somewhat delicate approximations.” Among those approximations is that the initial state is coupled to a large number of final states with similar energies. For a treatment of systems decaying into small numbers of final states and the attendant failure of the exponential “law,” see, for example Dittes (2000). Also of interest is Gaemers and Visser (1988).

In fact, there are many simple physical systems which instead display hyperbolic decay laws. Typical examples are those which involve excitation of pairs in the medium, which then recombine to emit light. This naturally gives decay laws which one would expect classically to obey  $dN/dt = -\lambda N^2$ . Note that this is a purely classical result and does not require coherent effects between excited states, which would also be expected to give the same decay law. For a review of condensed matter analogs of hyperbolic delayed luminescence (DL) in living things, see Scordino et al. (2000). Interestingly, in systems like CdS, the hyperbolic DL depends strongly on the size of the grains, and in the nematic liquid crystal 4-methoxybenzylidene-4-*n*-butylaniline (MBBA) it is present in the crystalline form, but disappears on melting (Scordino et al. 2000). In other words, the character of DL is not simply a matter of chemical composition, but can depend strongly on the form the material takes. Interestingly, the trend seems to be towards higher DL in systems exhibiting higher degrees of structure, a fact which seems relevant for biological systems.

Approximately hyperbolic decay laws also arise in correlated many-soliton states (Brizhik et al. 2001), which again may be relevant for biological systems.

It should be noted that, on general grounds, strict exponential decay is impossible in quantum mechanics. Khalifin (1958) showed as long ago as 1958 that the Paley–Wiener theorem, together with analyticity, forbids exponential decay at large times.

There is also a simple physical argument. Going to the energy representation from the time representation [i.e., taking a Fourier transform of the amplitude, which would be proportional to  $\exp(-\frac{1}{2}\lambda t)$ ], one finds a Lorentzian, which immediately gives two problems: the tail goes to infinity, so that a system with an initially finite energy could be found to have arbitrarily large energy, and the Lorentzian (squared) does not have a finite integral, so that the probability for any given energy interval is zero.

There is a theorem (Khalfin 1968) that  $P(t)$ , the probability that a system with a finite mean energy remains excited—that is, the survival probability—must satisfy  $dP(t)/dt = 0$  at  $t = 0$ , a property shared by neither the exponential nor hyperbolic decays, so they must at best be approximations where they do seem to work. This can also be seen directly from the textbook “derivation” of exponential decay before any approximations are made.

Thus we see that a strictly exponential decay law fails at both long and short times.

For the hyperbolic decay law, the same normalization problem in the energy representation is clearly present: the integral of  $1/(t - a)$  from zero to infinity is infinite, and again we find that a strictly hyperbolic decay law is unphysical, so nonlinear rate equations of this type must be approximations.

Weron and Weron (1984) have argued for a survival probability in general (for the cases where one might otherwise derive, with approximations, an exponential decay) of the form  $\exp(-t^\alpha)$ , where  $\alpha > 0$  and  $0 < \alpha < 1$ . It is even possible to have oscillations in decay (Giacosa and Pagliara 2012).

Since one in general expects deviations from either exponential or hyperbolic decays, any and all experimental data are welcome—the form of a decay law can, and indeed must, be more complicated than a simple exponential or hyperbolic decay law, however well those models may fit data over a restricted time interval.

In the following sections we discuss some nonlinear rate equations that have been successfully employed for diverse biosystems.

### Dynamics behind some simple nonlinear rate equations

Here we discuss some simple nonlinear rate equations and the dynamical reasons behind them. The linear rate equation where the rate is proportional to the number itself leads of course to an exponential growth or an exponential decay. However, as in all practical systems, some nonlinearity is

bound to be present, giving rise to a nonexponential in time behavior.

The best studied example is that of the laser. Here, if the mean photon number rate equation were linear, the number of laser photons would increase exponentially. Of course, that cannot be, otherwise we would need an infinite source of energy, hence there must be some dynamical mechanism to saturate the number. The solution to this problem was first given by Willis E. Lamb. The famous Lamb equation for light intensity  $I(t)$  (Mandel and Wolfe 1995) may be written as

$$\frac{dI(t)}{dt} = +\nu[a - I(t)]I(t). \quad (7)$$

The second term on the right-hand side of Eq. (7) arises dynamically through the creation and annihilation of two photons at a time, just as the first term is related to the creation and annihilation of single photons. The parameter  $a$  is called the pump parameter, and its sign is crucial in determining the steady-state value of  $I$ .

If  $a \leq 0$ , the steady-state value of  $I$  [determined by the vanishing of the left-hand side of Eq. (7)] is  $I_{SS} \rightarrow 0$ . Physically, for negative pump parameters, there is no laser activity. On the other hand though, for  $a > 0$ ,  $I_{SS} \rightarrow a$  and hence the laser intensity increases linearly with  $a$ .

Also, the innocent-looking Eq. (7) has buried in it a (second-order) phase transition wherein  $a$  acts as the order parameter. This is easily seen by considering  $I_{SS}$  as a function of  $a$ .  $I_{SS}$  is continuous at  $a = 0$ , but its derivative is not.

A simple model for a plethora of physical processes such as mean photon number, intensity, mass growth, magnetization, etc. is provided by analogs to Eq. (7) where the parameters  $\nu$  and  $a$  have different physical significance and their signs play a crucial role in determining the fate of each physical system.

As discussed above in “[Electric fields and frequencies in biosystems](#),” the frequency of a mode in a biological cell is inversely proportional to the length in accordance with Eq. (1). If the cell geometry fluctuates via the length scale  $L$ , then the frequency of the photon oscillator will be modulated. Because of such modulation, the cell cavity will emit or absorb two photons at a time, thus leading to the above rate equation.

Similar (nonlinear) rate equations must exist for any living system (e.g., for its size, cell number, etc.) where their growth may be initially rapid but eventually cease, resulting in a limiting value (such as the maximum size). In the next section, we discuss the concrete case of biophoton emission from soybeans as well as the rate equation governing the growth in mass of said soybeans.

### Growth of biological mass

The growth of biological soybean sprouts plus roots has been written as the solution of the logistic differential equation (Verhulst 1845, 1847)

$$\frac{dM}{dt} = v \left( M - \frac{M^2}{M_\infty} \right). \quad (8)$$

In this model, for low mass the growth rate is proportional to mass, since the nutrients feeding the bean sprout are proportional to the mass. For higher mass the loss rate from soybean waste emission is proportional to the square of the mass because of second-order reaction kinetics.

With an initial mass  $M_0$ , the solution of Eq. (8) is given by

$$M(t) = \frac{M_0 M_\infty}{M_0 + (M_\infty - M_0)e^{-vt}}, \quad (9)$$

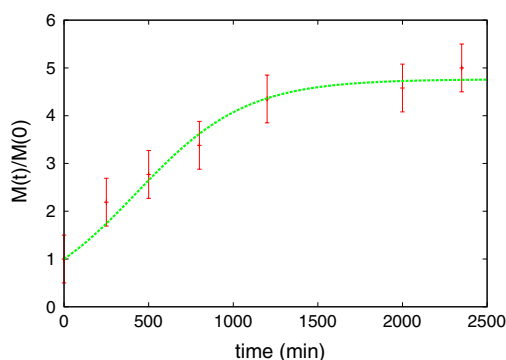
where  $M_\infty$  is the final steady-state value of  $M(t \rightarrow \infty)$ . In a measurement (Popp 1988) of the growth of bean sprout mass, one had

$$\begin{aligned} v &= 3.1 \times 10^{-3} \text{ min}, \\ M_0 &= 1.3 \text{ g}, \\ M_\infty &= 6.19 \text{ g}. \end{aligned} \quad (10)$$

A comparison between the theoretical Eq. (9) and the experimental data on growing soybean sprouts is shown in Fig. 1. The agreement between theory and experiment is satisfactory.

### Photons and cancer cell growth

It is well documented that, at high cell densities, large differences in photon emission exist between normal cells and tumor cells originating from the same parental tissue



**Fig. 1** Data points exhibited from Popp et al.'s (1988) experiment showing the growing mass of soybeans as a function of time. The continuous curve is obtained using logistic Eqs. (9) and (10)

(Schamhart and van Wijk 1987; Takeda et al. 1998). If the intrinsic rate of growth  $v$  depends explicitly on time, one finds a rate-modulated logistic Eq. (8) of the form

$$\frac{dM}{dt} = v(t) \left( M - \frac{M^2}{M_\infty} \right). \quad (11)$$

The solution of Eq. (11) is

$$\begin{aligned} \eta(t) &= \int_0^t v(s) ds, \\ M(t) &= \frac{M_0 M_\infty}{M_0 + (M_\infty - M_0)e^{-\eta(t)}}. \end{aligned} \quad (12)$$

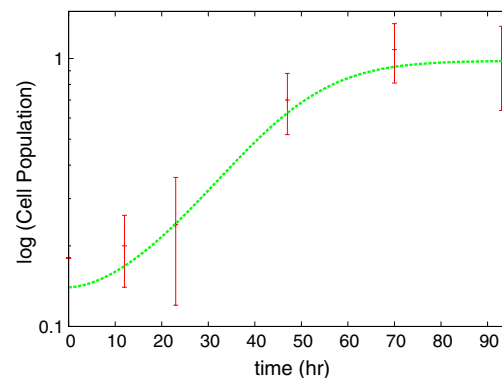
The growth curves of esophageal cancer cell (TE9) mass have been carefully studied (Takeda et al. 1998). We have fit the growth curves employing Eq. (12) with the stretched exponential form

$$\eta(t) = (t/\tau)^r, \quad \tau \approx 6.31 \text{ h} \quad \text{and} \quad r \approx 1.75. \quad (13)$$

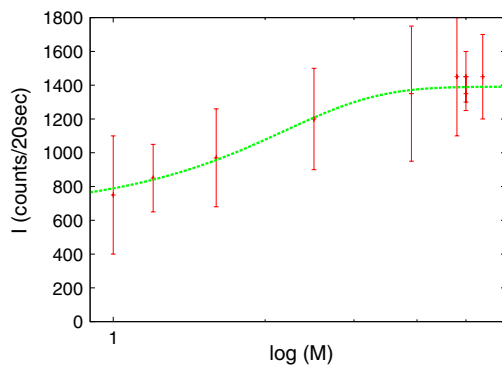
The results are plotted in Fig. 2. The agreement with the stretched exponential form in Eqs. (12) and (13) is satisfactory.

The biophoton emission intensity from cancer cells as a function of time has also been measured (Takeda et al. 1998). One may thus obtain the biophoton intensity as a function of the number of cancer cells. A plot of the results is shown in Fig. 3. As the number of cancer cells increase, so does the biophoton emission rate of *each cell*.

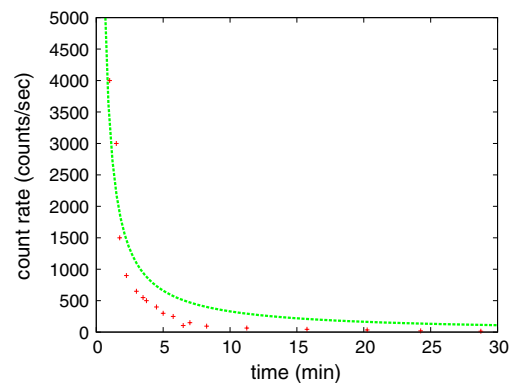
The large differences between the (1) growth curves for cancer (versus normal) cells with respect to mass, as well as (2) the biophoton emission intensity in the two cases give us greater insight towards understanding the fundamental mechanisms behind them. Of course, despite recent advances achieved from the chemical side (Rastogi and Pospisil 2010; Cifra and Pospisil 2014; Pospisil et al. 2014), much remains unknown about the physics of the problem such as the possibility of a phase transition that needs to be explored and augmented.



**Fig. 2** Fit curve of the stretched exponential theoretical Eqs. (12) and (13) to experimental data on growth of cancerous cell populations in arbitrary units as a function of time



**Fig. 3** Theoretical fit of Eqs. (12) and (13) to Takeda et al.'s experimental data. The biophoton emission rate is depicted as a function of the logarithm of the growing cancerous cell mass  $M$  in arbitrary units



**Fig. 4** Biophoton decay rate versus time for cells in *Bryophyllum daigremontianum*. The curve is the result of the fit of Eq. (16) to experimental data

### Luminescent decay

A special case of Eq. (7) leads to a hyperbolic decay. Let us substitute

$$a = 0; \quad \nu = \frac{\nu_0}{N(0)}, \quad (14)$$

so that Eq. (7) takes the form

$$\frac{dN(t)}{dt} = -\left(\frac{\nu_0}{N(0)}\right)N^2(t), \quad (15)$$

whose solution is

$$N(t) = \frac{N(0)}{1 + \nu_0 t}. \quad (16)$$

A physical example (Popp et al. 1981) of biophoton decay obeying the hyperbolic law is provided in Fig. 4.

An extension of Eq. (16) to a fractional power in time, in analogy to that of the stretched exponential as in the last section [see Eq. (13)], i.e.,

$$M(t) = \frac{M(0)}{[1 + \nu_0(t/t_0)^k]}, \quad (17)$$

has been made in Scholz et al. (1988), where light-stimulated biophoton reemission from normal and cancerous cells are compared. They find a considerable difference in the value of the parameter  $k$  for the two cases.

### Conclusions

An important element in analysis of biophotons concerns the magnitudes of the electric fields. Our estimates for the average electric fields in a cell and nucleus—considered as a cavity—show that these fields are quite large, see “Electric fields and frequencies in biosystems.” For example, the

field due to one such biophoton in a nucleus is only slightly smaller than the breakdown field in humid conditions.

In the rest of the paper we have considered various rate equations of particular relevance to biological systems and provided physical reasons behind those equations where possible; see “Dynamics behind some simple nonlinear rate equations,” “Growth of biological mass,” and “Luminescent decay.” Theoretical arguments were presented to establish that the commonly employed exponential law is not strictly tenable. A case of practical interest, namely the delineation between the behavior of normal versus cancer cells, is illustrated through differences in the values of parameters in their rate equations, see “Photons and cancer cell growth.”

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